

Curriculum Framework

Mathematics

Elementary

- **CF.MA.K-5.I** - Patterns, Relationships and Functions
 - **CF.MA.K-5.I.1** - Students recognize similarities and generalize patterns, use patterns to create models and make predictions, describe the nature of patterns and relationships, and construct representations of mathematical relationships.
 - **CF.MA.K-5.I.1.1** - Recognize, describe and extend numerical and geometric patterns.
 - **CF.MA.K-5.I.1.2** - Represent and record patterns and relationships in a variety of ways including tables, charts and pictures.
 - **CF.MA.K-5.I.1.3** - Use patterns to describe real-world phenomena.
 - **CF.MA.K-5.I.1.4** - Explore various types of numeric and geometric patterns (repeating, growing, shrinking).
 - **CF.MA.K-5.I.1.5** - Apply their experiences with patterns to help solve problems and explore new content.
 - **CF.MA.K-5.I.2** - Students describe the relationships among variables, predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change.
 - **CF.MA.K-5.I.2.1** - Recognize change and variability when it occurs in a variety of settings.
 - **CF.MA.K-5.I.2.2** - Recognize that change is often predictable, but variable, and that patterns emerge that help to describe the change.
 - **CF.MA.K-5.I.2.3** - Explore change, and realize that changes are frequently interdependent.
 - **CF.MA.K-5.I.2.4** - Use tables, charts, open sentences and hands-on models to represent change and variability.
 - **CF.MA.K-5.I.2.5** - Begin to describe and differentiate between types of relationships, especially repeating, growing and shrinking patterns.
 - **CF.MA.K-5.I.2.6** - Explore variability and change in a variety of contexts, investigations and problems.
- **CF.MA.K-5.II** - Geometry and Measurement
 - **CF.MA.K-5.II.1** - Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes, identify properties and describe relationships among shapes.
 - **CF.MA.K-5.II.1.1** - Recognize and name familiar shapes in one, two and three dimensions such as lines, rectangles and spheres and informally discuss the shape of a graph.
 - **CF.MA.K-5.II.1.2** - Describe the attributes of familiar shapes.
 - **CF.MA.K-5.II.1.3** - Compare, sort and classify familiar shapes.
 - **CF.MA.K-5.II.1.4** - Draw and build familiar shapes.
 - **CF.MA.K-5.II.1.5** - Explore ways to combine, dissect and transform shapes.
 - **CF.MA.K-5.II.1.6** - Recognize parallel and perpendicular line segments and figures that have similarity and/or congruence.
 - **CF.MA.K-5.II.1.7** - Use shape, shape properties and shape relationships to describe the physical world and to solve problems.
 - **CF.MA.K-5.II.2** - Students identify locations of objects, identify location relative to other objects, and describe the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.
 - **CF.MA.K-5.II.2.1** - Locate and describe objects in terms of their position, including front, back, inside, outside, right, left, over, under, next to, between and locations on the number line, on a coordinate graph and on a map.
 - **CF.MA.K-5.II.2.2** - Locate and describe objects in terms of their orientation, direction and relative position, including up, down, front, back, N- S- E- W, flipped, turned, translated; recognize symmetrical objects and identify their lines of symmetry.
 - **CF.MA.K-5.II.2.3** - Explore what happens to the size, shape and position of an object after sliding, flipping, turning, enlarging or reducing it.
 - **CF.MA.K-5.II.2.4** - Michigan has no content for this Benchmark at this grade cluster.
 - **CF.MA.K-5.II.2.5** - Use concepts of position, direction and orientation to describe the physical world and to solve problems.
 - **CF.MA.K-5.II.3** - Students compare attributes of two objects, or of one object with a standard (unit), and analyze situations to determine what measurement(s) should be made and to what level of precision.
 - **CF.MA.K-5.II.3.1** - Compare attributes of objects; develop standard units of measurement; and select and use standard tools for measurement.
 - **CF.MA.K-5.II.3.2** - Identify the attribute to be measured and select the appropriate unit of measurement for length, mass (weight), area, perimeter, capacity, time, temperature and money.
 - **CF.MA.K-5.II.3.3** - Develop strategies for estimating measures and compare the estimates to the results of the measurement; decide if an estimate is “a good estimate.”
 - **CF.MA.K-5.II.3.4** - Explain the meaning of measurements and recognize that the number of units it takes to measure an object is related to the size of the unit.
 - **CF.MA.K-5.II.3.5** - Explore scale drawings, models and maps and relate them to measurements of real objects.
 - **CF.MA.K-5.II.3.6** - Apply measurement to describe the real world and to solve problems.
- **CF.MA.K-5.III** - Data Analysis and Statistics
 - **CF.MA.K-5.III.1** - Students collect and explore data, organize data into a useful form, and develop skill in representing and reading data displayed in different formats.
 - **CF.MA.K-5.III.1.1** - Collect and explore data through counting, measuring and conducting surveys and experiments.

- **CF.MA.K-5.III.1.2** - Organize data using concrete objects, pictures, tallies, tables, charts, diagrams and graphs.
- **CF.MA.K-5.III.1.3** - Present data using a variety of appropriate representations and explain the meaning of the data.
- **CF.MA.K-5.III.1.4** - Identify what data are needed to answer a particular question or solve a given problem, and design and implement strategies to obtain, organize and present those data.
- **CF.MA.K-5.III.2** - Students examine data and describe characteristics of a distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively.
 - **CF.MA.K-5.III.2.1** - Read and explain data they have collected and organized themselves and progress to reading data from other sources.
 - **CF.MA.K-5.III.2.2** - Describe the shape of the data using informal language.
 - **CF.MA.K-5.III.2.3** - Draw, explain and justify conclusions, such as trends based on data.
 - **CF.MA.K-5.III.2.4** - Raise and answer questions about the source, collection, organization and presentation of data, as well as the conclusions drawn from the data; explore biases in the data.
 - **CF.MA.K-5.III.2.5** - Formulate questions and problems and gather and interpret data to answer those questions.
- **CF.MA.K-5.III.3** - Students draw defensible inferences about unknown outcomes, make predictions, and identify the degree of confidence they have in their predictions.
 - **CF.MA.K-5.III.3.1** - Make and test hypotheses.
 - **CF.MA.K-5.III.3.2** - Conduct surveys, samplings and experiments to solve problems and answer questions of interest to them.
 - **CF.MA.K-5.III.3.3** - Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.
 - **CF.MA.K-5.III.3.4** - Make and explain predictions based on data.
 - **CF.MA.K-5.III.3.5** - Make predictions to answer questions and solve problems.
- **CF.MA.K-5.IV** - Number Sense and Numeration
 - **CF.MA.K-5.IV.1** - Students experience counting and measuring activities to develop intuitive sense about numbers, develop understanding about properties of numbers, understand the need for and existence of different sets of numbers, and investigate properties of special numbers.
 - **CF.MA.K-5.IV.1.1** - Develop an understanding of whole numbers and read, write and count using whole numbers; investigate basic concepts of fractions and decimals.
 - **CF.MA.K-5.IV.1.2** - Investigate and develop an understanding of the base-10 place-value system.
 - **CF.MA.K-5.IV.1.3** - Develop an understanding of the properties of numbers (e.g., order) and of the properties of the special numbers 0 and 1.
 - **CF.MA.K-5.IV.1.4** - Apply their understanding of number systems to model and solve problems.
 - **CF.MA.K-5.IV.2** - Students recognize that numbers are used in different ways such as counting, measuring, ordering and estimating, understand and produce multiple representations of a number, and translate among equivalent representations.
 - **CF.MA.K-5.IV.2.1** - Represent whole numbers, fractions and decimals using concrete, pictorial and symbolic representations.
 - **CF.MA.K-5.IV.2.2** - Explore and recognize different representations for the same number and explain why they are the same.
 - **CF.MA.K-5.IV.2.3** - Investigate ways numbers are used (e.g., counting, ordering, naming, locating, measuring).
 - **CF.MA.K-5.IV.2.4** - Develop strategies for estimating quantity and evaluate the reasonableness of their estimates.
 - **CF.MA.K-5.IV.2.5** - Select appropriate numbers and representations in order to solve problems.
 - **CF.MA.K-5.IV.3** - Students investigate relationships such as equality, inequality, inverses, factors and multiples, and represent and compare very large and very small numbers.
 - **CF.MA.K-5.IV.3.1** - Compare and order numbers using “equal,” “less than” or “greater than.”
 - **CF.MA.K-5.IV.3.2** - Use part-whole relationships to explore numbers, develop number concepts and understand computation.
 - **CF.MA.K-5.IV.3.3** - Classify numbers as even or odd and explore concepts of factors and multiples.
 - **CF.MA.K-5.IV.3.4** - Michigan has no content for this Benchmark at this grade cluster.
 - **CF.MA.K-5.IV.3.5** - Apply their understanding of number relationships in solving problems.
- **CF.MA.K-5.V** - Numerical and Algebraic Operations and Analytical Thinking
 - **CF.MA.K-5.V.1** - Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems.
 - **CF.MA.K-5.V.1.1** - Use manipulatives to model operations with numbers; develop their own methods of recording operations; and relate their models and recordings to standard symbolic expressions and algorithms.
 - **CF.MA.K-5.V.1.2** - Develop and apply the appropriate method of computation from among mental computation, estimation, paper-and-pencil or calculators; explain why they are choosing a method and how they know which operations to perform in a given situation.
 - **CF.MA.K-5.V.1.3** - Explore properties of operations (e.g., commutative and distributive properties) and give examples of how they use those properties.
 - **CF.MA.K-5.V.1.4** - Apply operations efficiently and accurately in solving problems.
 - **CF.MA.K-5.V.2** - Students analyze problems to determine an appropriate process for solution, and use algebraic notations to model or represent problems. (Algebraic and Analytic Thinking)
 - **CF.MA.K-5.V.2.1** - Write and solve open sentences (e.g., $__ + __ = 5$) and write stories to fit the open sentence.

- **CF.MA.K-5.V.2.2** - Explore algebraic concepts with manipulatives such as balance scales, tables of input and output, and pictorial representations of problems.
- **CF.MA.K-5.V.2.3** - Find replacements for the variable(s) in open sentences.
- **CF.MA.K-5.V.2.4** - Use analytic thinking to describe situations and solve problems.
- **CF.MA.K-5.V.2.5** - Michigan has no content for this Benchmark at this grade cluster.
- **CF.MA.K-5.VI** - Probability and Discrete Mathematics
 - **CF.MA.K-5.VI.1** - Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available, and make critical judgments about claims that are made in probabilistic situations.
 - **CF.MA.K-5.VI.1.1** - Explain the difference between chance and certainty and give examples to illustrate their understanding.
 - **CF.MA.K-5.VI.1.2** - Compare events and describe them as “more likely” or “less likely” and use the language of fractions to describe simple probabilities.
 - **CF.MA.K-5.VI.1.3** - Conduct experiments with concrete objects to explore concepts and develop an intuitive understanding of how the conditions of the experiment can affect the outcome.
 - **CF.MA.K-5.VI.1.4** - Conduct experiments, record the outcomes, examine those outcomes to determine if they make sense and search for explanations of the outcomes.
 - **CF.MA.K-5.VI.1.5** - Conduct probability experiments and simulations to model and solve problems.
 - **CF.MA.K-5.VI.2** - Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying, and analyze ideas like recurrence relations, induction, iteration, and algorithm design.
 - **CF.MA.K-5.VI.2.1** - Use manipulatives and diagrams to explore problems involving counting and arranging objects.
 - **CF.MA.K-5.VI.2.2** - Explore sets and set relationships by sorting and classifying objects.
 - **CF.MA.K-5.VI.2.3** - Explore situations in which they model and trace paths using figures consisting of vertices connected by edges.
 - **CF.MA.K-5.VI.2.4** - Explore now-next patterns.
 - **CF.MA.K-5.VI.2.5** - Explore, develop and invent their own algorithms to accomplish a task or to solve numerical problems.
 - **CF.MA.K-5.VI.2.6** - Use discrete mathematics concepts as described above to model situations and solve problems; and look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems) and decide upon a best solution (optimization problems).

- **CF.MA.6-8.I** - Patterns, Relationships and Functions
 - **CF.MA.6-8.I.1** - Students recognize similarities and generalize patterns, use patterns to create models and make predictions, describe the nature of patterns and relationships, and construct representations of mathematical relationships.
 - **CF.MA.6-8.I.1.1** - Describe, analyze and generalize patterns arising in a variety of contexts and express them in general terms.
 - **CF.MA.6-8.I.1.2** - Represent and record patterns in a variety of ways including tables, charts and graphs, and translate between various representations.
 - **CF.MA.6-8.I.1.3** - Use patterns and their generalizations to make and justify inferences and predictions.
 - **CF.MA.6-8.I.1.4** - Explore and describe visual and numeric patterns, including linear expressions, near-linear patterns and symmetric and spatial patterns.
 - **CF.MA.6-8.I.1.5** - Use patterns and generalizations to solve problems and explore new content.
 - **CF.MA.6-8.I.2** - Students describe the relationships among variables, predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change.
 - **CF.MA.6-8.I.2.1** - Identify and describe the nature of change; recognize change in more abstract and complex situations and explore different kinds of change and patterns of variation.
 - **CF.MA.6-8.I.2.2** - Connect an initial state to a final state and generalize a rule that describes a pattern of change.
 - **CF.MA.6-8.I.2.3** - Begin to investigate applications in bivariate data and linear relationships and explore questions of what will happen to one quantity if another variable is changed.
 - **CF.MA.6-8.I.2.4** - Represent variability or change by ordered pairs, tables, graphs and equations.
 - **CF.MA.6-8.I.2.5** - Differentiate between functions and relationships such as linear vs. not linear or continuous vs. non-continuous.
 - **CF.MA.6-8.I.2.6** - Continue to explore relationships arising from interesting contexts and use variables and relationships to solve mathematical problems.
- **CF.MA.6-8.II** - Geometry and Measurement
 - **CF.MA.6-8.II.1** - Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes, identify properties and describe relationships among shapes.
 - **CF.MA.6-8.II.1.1** - Distinguish among shapes and differentiate between examples and non-examples of shapes based on their properties; generalize about shapes of graphs and data distributions.
 - **CF.MA.6-8.II.1.2** - Generalize the characteristics of shapes and apply their generalizations to classes of shapes.
 - **CF.MA.6-8.II.1.3** - Derive generalizations about shapes and apply those generalizations to develop classifications of familiar shapes.
 - **CF.MA.6-8.II.1.4** - Construct familiar shapes using coordinates, appropriate tools (including technology), sketching and drawing two- and three-dimensional shapes.
 - **CF.MA.6-8.II.1.5** - Combine, dissect and transform shapes.
 - **CF.MA.6-8.II.1.6** - Generalize about the common properties of similar, congruent, parallel and perpendicular shapes and verify their generalizations informally.
 - **CF.MA.6-8.II.1.7** - Use shape, shape properties and shape relationships to describe the physical world and to solve problems.
 - **CF.MA.6-8.II.2** - Students identify locations of objects, identify location relative to other objects, and describe the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.
 - **CF.MA.6-8.II.2.1** - Locate and describe objects in terms of their position, including compass directions, Cartesian coordinates, latitude and longitude and midpoints.
 - **CF.MA.6-8.II.2.2** - Locate and describe objects in terms of their orientation and relative position, including coincident, collinear, parallel, perpendicular; differentiate between fixed (e.g., N- S- E- W) and relative (e.g., right-left) orientations; recognize and describe examples of bilateral and rotational symmetry.
 - **CF.MA.6-8.II.2.3** - Describe translations, reflections, rotations and dilations using the language of transformations and employ transformations to verify congruence of figures.
 - **CF.MA.6-8.II.2.4** - Locate the position of points or objects described by two or more conditions; locate all the points (locus) that satisfy a given condition.
 - **CF.MA.6-8.II.2.5** - Use concepts of position, direction and orientation to describe the physical world and to solve problems.
 - **CF.MA.6-8.II.3** - Students compare attributes of two objects, or of one object with a standard (unit), and analyze situations to determine what measurement(s) should be made and to what level of precision.

- **CF.MA.6-8.II.3.1** - Select and use appropriate tools; measure objects using standard units in both the metric and common systems and measure angles in degrees.
- **CF.MA.6-8.II.3.2** - Identify the attribute to be measured and select the appropriate unit of measurement for length, mass (weight), time, temperature, perimeter, area, volume and angle.
- **CF.MA.6-8.II.3.3** - Estimate measures with a specified degree of accuracy and decide if an estimate or a measurement is “close enough.”
- **CF.MA.6-8.II.3.4** - Interpret measurements and recognize that two objects may have the same measurement on one attribute (e.g., area) but not necessarily on another (e.g., perimeter).
- **CF.MA.6-8.II.3.5** - Use proportional reasoning and indirect measurements to draw inferences.
- **CF.MA.6-8.II.3.6** - Apply measurement to describe the real world and to solve problems.
- **CF.MA.6-8.III** - Data Analysis and Statistics
 - **CF.MA.6-8.III.1** - Students collect and explore data, organize data into a useful form, and develop skill in representing and reading data displayed in different formats.
 - **CF.MA.6-8.III.1.1** - Collect and explore data through observation, measurement, surveys, sampling techniques and simulations.
 - **CF.MA.6-8.III.1.2** - Organize data using tables, charts, graphs, spreadsheets and data bases.
 - **CF.MA.6-8.III.1.3** - Present data using a variety of appropriate representations and explain why one representation is preferred over another or how a particular representation may bias the presentation.
 - **CF.MA.6-8.III.1.4** - Identify what data are needed to answer a particular question or solve a given problem, and design and implement strategies to obtain, organize and present those data.
 - **CF.MA.6-8.III.2** - Students examine data and describe characteristics of a distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively.
 - **CF.MA.6-8.III.2.1** - Critically read data from tables, charts or graphs and explain the source of the data and what the data represent.
 - **CF.MA.6-8.III.2.2** - Describe the shape of a data distribution and identify the center, the spread, correlations and any outliers.
 - **CF.MA.6-8.III.2.3** - Draw, explain and justify conclusions based on data.
 - **CF.MA.6-8.III.2.4** - Critically question the sources of data; the techniques used to collect, organize and present data; the inferences drawn from the data; and the possible sources of bias in the data or their presentation.
 - **CF.MA.6-8.III.2.5** - Formulate questions and problems and gather and interpret data to answer those questions.
 - **CF.MA.6-8.III.3** - Students draw defensible inferences about unknown outcomes, make predictions, and identify the degree of confidence they have in their predictions.
 - **CF.MA.6-8.III.3.1** - Make and test hypotheses.
 - **CF.MA.6-8.III.3.2** - Design experiments to model and solve problems using sampling, simulations and controlled investigations.
 - **CF.MA.6-8.III.3.3** - Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.
 - **CF.MA.6-8.III.3.4** - Make predictions and decisions based on data, including interpolations and extrapolations.
 - **CF.MA.6-8.III.3.5** - Employ investigations, mathematical models and simulations to make inferences and predictions to answer questions and solve problems.
- **CF.MA.6-8.IV** - Number Sense and Numeration
 - **CF.MA.6-8.IV.1** - Students experience counting and measuring activities to develop intuitive sense about numbers, develop understanding about properties of numbers, understand the need for and existence of different sets of numbers, and investigate properties of special numbers.
 - **CF.MA.6-8.IV.1.1** - Develop an understanding of integers and rational numbers and represent rational numbers in both fraction and decimal form.
 - **CF.MA.6-8.IV.1.2** - Extend their understanding of numeration systems to include decimal numeration, scientific numeration and non-decimal numeration systems.
 - **CF.MA.6-8.IV.1.3** - Develop an understanding of the properties of the integer and rational number systems (e.g., order, density) and of the properties of special numbers including 0, 1 and pi, and the additive and multiplicative inverses.
 - **CF.MA.6-8.IV.1.4** - Apply their understanding of number systems to model and solve mathematical and applied problems.
 - **CF.MA.6-8.IV.2** - Students recognize that numbers are used in different ways such as counting, measuring, ordering and estimating, understand and produce multiple representations of a number, and translate among equivalent representations.
 - **CF.MA.6-8.IV.2.1** - Give geometric representations of fractions, prime and composite numbers, triangular and square numbers, and other number concepts; represent rational numbers and integers on the number line.
 - **CF.MA.6-8.IV.2.2** - Recognize equivalent representations of a number, especially fractions, decimals and percents, and translate freely among representations.
 - **CF.MA.6-8.IV.2.3** - Distinguish between numbers that are used for counting, numbers that are used for ordering, numbers that are used for measuring and numbers that are used for naming.
 - **CF.MA.6-8.IV.2.4** - Develop and refine strategies for estimating quantities, including fractional quantities, and evaluate the reasonableness and appropriateness of their estimates.

- **CF.MA.6-8.IV.2.5** - Select appropriate representations for numbers, including integers and rational numbers, in order to simplify and solve problems.
- **CF.MA.6-8.IV.3** - Students investigate relationships such as equality, inequality, inverses, factors and multiples, and represent and compare very large and very small numbers.
 - **CF.MA.6-8.IV.3.1** - Compare and order integers and rational numbers using relations of equality and inequality.
 - **CF.MA.6-8.IV.3.2** - Express numerical comparisons as ratios and rates.
 - **CF.MA.6-8.IV.3.3** - Distinguish between prime and composite numbers; identify factors, multiples, common factors and multiples, and relatively prime numbers; and apply divisibility tests to numbers.
 - **CF.MA.6-8.IV.3.4** - Explain the meaning of powers and roots of numbers and use calculators to compute powers and square roots.
 - **CF.MA.6-8.IV.3.5** - Apply their understanding of number relationships in solving problems.
- **CF.MA.6-8.V** - Numerical and Algebraic Operations and Analytical Thinking
 - **CF.MA.6-8.V.1** - Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems.
 - **CF.MA.6-8.V.1.1** - Use manipulatives and diagrams to model operations and their inverses with integers and rational numbers and relate the models to their symbolic expressions.
 - **CF.MA.6-8.V.1.2** - Compute with integers, rational numbers and simple algebraic expressions using mental computation, estimation, calculators and paper-and-pencil; explain what they are doing and how they know which operations to perform in a given situation.
 - **CF.MA.6-8.V.1.3** - Describe the properties of operations with rationals and integers (e.g., closure; associative, commutative and distributive properties) and give examples of how they use those properties.
 - **CF.MA.6-8.V.1.4** - Efficiently and accurately apply operations with integers, rational numbers and simple algebraic expressions in solving problems.
 - **CF.MA.6-8.V.2** - Students analyze problems to determine an appropriate process for solution, and use algebraic notations to model or represent problems. (Algebraic and Analytic Thinking)
 - **CF.MA.6-8.V.2.1** - Read and write algebraic expressions; develop original examples expressed verbally and algebraically; simplify expressions and translate between verbal and algebraic expressions; and solve linear equations and inequalities.
 - **CF.MA.6-8.V.2.2** - Represent algebraic concepts with geometric models (e.g., algebra tiles), physical models (e.g., balance beam), tables and graphs; and write algebraic expressions to correspond to the multiple representations.
 - **CF.MA.6-8.V.2.3** - Solve linear equalities and inequalities using algebraic and geometric methods, and use the context of the problem to interpret and explain their solutions.
 - **CF.MA.6-8.V.2.4** - Analyze problems modeled by linear functions, determine strategies for solving the problems and evaluate the adequacy of the solutions in the context of the problems.
 - **CF.MA.6-8.V.2.5** - Explore problems that reflect the contemporary uses of mathematics in significant contexts and use the power of technology and algebraic and analytic reasoning to experience the ways mathematics is used in society.
 - **CF.MA.6-8.VI** - Probability and Discrete Mathematics
 - **CF.MA.6-8.VI.1** - Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available, and make critical judgments about claims that are made in probabilistic situations.
 - **CF.MA.6-8.VI.1.1** - Describe events as likely or unlikely and give qualitative and quantitative descriptions of the degree of likelihood.
 - **CF.MA.6-8.VI.1.2** - Describe probability as a measure of certainty ranging from 0 to 1 and conduct activities that allow them to express probabilities of simple events in mathematical terms.
 - **CF.MA.6-8.VI.1.3** - Conduct experiments and give examples to illustrate the difference between dependent and independent events.
 - **CF.MA.6-8.VI.1.4** - Explain the difference between probabilities determined from experiments or chance events (empirical) and probabilities derived mathematically (theoretical), and explain how the empirical probability changes for a large number of trials.
 - **CF.MA.6-8.VI.1.5** - Conduct probability experiments and simulations to model and solve problems.
 - **CF.MA.6-8.VI.2** - Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying, and analyze ideas like recurrence relations, induction, iteration, and algorithm design.
 - **CF.MA.6-8.VI.2.1** - Use manipulatives, diagrams and the fundamental theorem of counting to count permutations and combinations.
 - **CF.MA.6-8.VI.2.2** - Use sets and set relationships to explore and solve simple algebraic and geometric problems.
 - **CF.MA.6-8.VI.2.3** - Solve problems involving networks, for example planning delivery routes or counting paths between points.
 - **CF.MA.6-8.VI.2.4** - Explore recurrence relations and iterations.
 - **CF.MA.6-8.VI.2.5** - Continue to use manipulatives and drawings to model the concepts and procedures for the standard arithmetic algorithms, and develop and analyze their own and other students' algorithms to accomplish a task or solve a mathematical problem.
 - **CF.MA.6-8.VI.2.6** - Use discrete mathematics concepts as described above to model situations and solve problems; and look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems) and decide upon a best solution (optimization problems).

Curriculum Framework

Mathematics

High School

- **CF.MA.9-12.I** - Patterns, Relationships and Functions
 - **CF.MA.9-12.I.1** - Students recognize similarities and generalize patterns, use patterns to create models and make predictions, describe the nature of patterns and relationships, and construct representations of mathematical relationships.
 - **CF.MA.9-12.I.1.1** - Analyze and generalize mathematical patterns including sequences, series and recursive patterns.
 - **CF.MA.9-12.I.1.2** - Analyze, interpret and translate among representations of patterns including tables, charts, graphs, matrices and vectors.
 - **CF.MA.9-12.I.1.3** - Study and employ mathematical models of patterns to make inferences, predictions and decisions.
 - **CF.MA.9-12.I.1.4** - Explore patterns (graphic, numeric, etc.) characteristic of families of functions; explore structural patterns within systems of objects, operations or relations.
 - **CF.MA.9-12.I.1.5** - Use patterns and reasoning to solve problems and explore new content.
 - **CF.MA.9-12.I.2** - Students describe the relationships among variables, predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change.
 - **CF.MA.9-12.I.2.1** - Identify and describe the nature of change and begin to use the more formal language such as rate of change, continuity, limit, distribution and deviation.
 - **CF.MA.9-12.I.2.2** - Develop a mathematical concept of function and recognize that functions display characteristic patterns of change (e.g., linear, quadratic, exponential).
 - **CF.MA.9-12.I.2.3** - Expand their understanding of function to include non-linear functions, composition of functions, inverses of functions, and piecewise- and recursively-defined functions.
 - **CF.MA.9-12.I.2.4** - Represent functions using symbolism such as matrices, vectors and functional representation ($f(x)$).
 - **CF.MA.9-12.I.2.5** - Differentiate and analyze classes of functions including linear, power, quadratic, exponential, circular and trigonometric functions, and realize that many different situations can be modeled by a particular type of function.
 - **CF.MA.9-12.I.2.6** - Increase their use of functions and mathematical models to solve problems in context.
- **CF.MA.9-12.II** - Geometry and Measurement
 - **CF.MA.9-12.II.1** - Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes, identify properties and describe relationships among shapes.
 - **CF.MA.9-12.II.1.1** - Use shape to identify plane and solid figures, graphs, loci, functions and data distributions.
 - **CF.MA.9-12.II.1.2** - Determine necessary and sufficient conditions for the existence of a particular shape and apply those conditions to analyze shapes.
 - **CF.MA.9-12.II.1.3** - Use transformational, coordinate or synthetic methods to verify (prove) the generalizations they have made about properties of classes of shapes.
 - **CF.MA.9-12.II.1.4** - Draw and construct shapes in two and three dimensions and analyze and justify the steps of their constructions.
 - **CF.MA.9-12.II.1.5** - Study transformations of shapes using isometries, size transformations and coordinate mappings.
 - **CF.MA.9-12.II.1.6** - Compare and analyze shapes and formally establish the relationships among them, including congruence, similarity, parallelism, perpendicularity and incidence.
 - **CF.MA.9-12.II.1.7** - Use shape, shape properties and shape relationships to describe the physical world and to solve problems.
 - **CF.MA.9-12.II.2** - Students identify locations of objects, identify location relative to other objects, and describe the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.
 - **CF.MA.9-12.II.2.1** - Locate and describe objects in terms of their position, including polar coordinates, three-dimensional Cartesian coordinates, vectors and limits.
 - **CF.MA.9-12.II.2.2** - Locate and describe objects in terms of their orientation and relative position, including displacement (vectors), phase shift, maxima, minima and inflection points; give precise mathematical descriptions of symmetries.
 - **CF.MA.9-12.II.2.3** - Give precise mathematical descriptions of transformations and describe the effects of transformations on size, shape, position and orientation.
 - **CF.MA.9-12.II.2.4** - Describe the locus of a point by a rule or mathematical expression; trace the locus of a moving point.
 - **CF.MA.9-12.II.2.5** - Use concepts of position, direction and orientation to describe the physical world and to solve problems.
 - **CF.MA.9-12.II.3** - Students compare attributes of two objects, or of one object with a standard (unit), and analyze situations to determine what measurement(s) should be made and to what level of precision.
 - **CF.MA.9-12.II.3.1** - Select and use appropriate tools; make accurate measurements using both metric and common units, and measure angles in degrees and radians.

- **CF.MA.9-12.II.3.2** - Continue to make and apply measurements of length, mass (weight), time, temperature, area, volume, angle; classify objects according to their dimensions.
- **CF.MA.9-12.II.3.3** - Estimate measures with a specified degree of accuracy and evaluate measurements for accuracy, precision and tolerance.
- **CF.MA.9-12.II.3.4** - Interpret measurements and explain how changes in one measure may affect other measures.
- **CF.MA.9-12.II.3.5** - Use proportional reasoning and indirect measurements, including applications of trigonometric ratios, to measure inaccessible distances and to determine derived measures such as density.
- **CF.MA.9-12.II.3.6** - Apply measurement to describe the real world and to solve problems.
- **CF.MA.9-12.III** - Data Analysis and Statistics
 - **CF.MA.9-12.III.1** - Students collect and explore data, organize data into a useful form, and develop skill in representing and reading data displayed in different formats.
 - **CF.MA.9-12.III.1.1** - Collect and explore data through observation, measurement, surveys, sampling techniques and simulations.
 - **CF.MA.9-12.III.1.2** - Organize data using tables, charts, graphs, spreadsheets and data bases.
 - **CF.MA.9-12.III.1.3** - Present data using the most appropriate representation and give a rationale for their choice; show how certain representations may skew the data or bias the presentation.
 - **CF.MA.9-12.III.1.4** - Identify what data are needed to answer a particular question or solve a given problem and design and implement strategies to obtain, organize and present those data.
 - **CF.MA.9-12.III.2** - Students examine data and describe characteristics of a distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively.
 - **CF.MA.9-12.III.2.1** - Critically read data from tables, charts or graphs and explain the source of the data and what the data represent.
 - **CF.MA.9-12.III.2.2** - Describe the shape of a data distribution and determine measures of central tendency, variability and correlation.
 - **CF.MA.9-12.III.2.3** - Use the data and their characteristics to draw and support conclusions.
 - **CF.MA.9-12.III.2.4** - Critically question the sources of data; the techniques used to collect, organize and present data; the inferences drawn from the data; and the sources of bias and measures taken to eliminate such bias.
 - **CF.MA.9-12.III.2.5** - Formulate questions and problems and gather and interpret data to answer those questions.
 - **CF.MA.9-12.III.3** - Students draw defensible inferences about unknown outcomes, make predictions, and identify the degree of confidence they have in their predictions.
 - **CF.MA.9-12.III.3.1** - Make and test hypotheses.
 - **CF.MA.9-12.III.3.2** - Design investigations to model and solve problems; also employ confidence intervals and curve fitting in analyzing the data.
 - **CF.MA.9-12.III.3.3** - Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.
 - **CF.MA.9-12.III.3.4** - Make predictions and decisions based on data, including interpolations and extrapolations.
 - **CF.MA.9-12.III.3.5** - Employ investigations, mathematical models, and simulations to make inferences and predictions to answer questions and solve problems.
- **CF.MA.9-12.IV** - Number Sense and Numeration
 - **CF.MA.9-12.IV.1** - Students experience counting and measuring activities to develop intuitive sense about numbers, develop understanding about properties of numbers, understand the need for and existence of different sets of numbers, and investigate properties of special numbers.
 - **CF.MA.9-12.IV.1.1** - Develop an understanding of irrational, real and complex numbers.
 - **CF.MA.9-12.IV.1.2** - Use the $(a+bi)$ and polar forms of complex numbers.
 - **CF.MA.9-12.IV.1.3** - Develop an understanding of the properties of the real and complex number systems and of the properties of special numbers including π , i , e , and conjugates.
 - **CF.MA.9-12.IV.1.4** - Apply their understanding of number systems to model, and solve mathematical and applied problems.
 - **CF.MA.9-12.IV.2** - Students recognize that numbers are used in different ways such as counting, measuring, ordering and estimating, understand and produce multiple representations of a number, and translate among equivalent representations.
 - **CF.MA.9-12.IV.2.1** - Give decimal representations of rational and irrational numbers and coordinate and vector representations of complex numbers.
 - **CF.MA.9-12.IV.2.2** - Develop an understanding of more complex representations of numbers, including exponential and logarithmic expressions, and select an appropriate representation to facilitate problem solving.
 - **CF.MA.9-12.IV.2.3** - Determine when to use rational approximations and the exact values of numbers such as e , π and the irrational.
 - **CF.MA.9-12.IV.2.4** - Apply estimation in increasingly complex situations.
 - **CF.MA.9-12.IV.2.5** - Select appropriate representations for numbers, including representations of rational and irrational numbers and coordinate and vector representations of complex numbers, in order to simplify and solve problems.
 - **CF.MA.9-12.IV.3** - Students investigate relationships such as equality, inequality, inverses, factors and multiples, and represent and compare very large and very small numbers.
 - **CF.MA.9-12.IV.3.1** - Compare and order real numbers and compare rational approximations to exact values.

- **CF.MA.9-12.IV.3.2** - Express numerical comparisons as ratios and rates.
- **CF.MA.9-12.IV.3.3** - Extend the relationships of primes, factors, multiples and divisibility in an algebraic setting.
- **CF.MA.9-12.IV.3.4** - Express number relationships using positive and negative rational exponents, logarithms and radicals.
- **CF.MA.9-12.IV.3.5** - Apply their understanding of number relationships in solving problems.
- **CF.MA.9-12.V** - Numerical and Algebraic Operations and Analytical Thinking
 - **CF.MA.9-12.V.1** - Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems.
 - **CF.MA.9-12.V.1.1** - Present and explain geometric and symbolic models for operations with real and complex numbers and algebraic expressions.
 - **CF.MA.9-12.V.1.2** - Compute with real numbers, complex numbers, algebraic expressions, matrices and vectors using technology and, for simple instances, with paper- and-pencil algorithms.
 - **CF.MA.9-12.V.1.3** - Describe the properties of operations with numbers, algebraic expressions, vectors and matrices, and make generalizations about the properties of given mathematical systems.
 - **CF.MA.9-12.V.1.4** - Efficiently and accurately apply operations with real numbers, complex numbers, algebraic expressions, matrices and vectors in solving problems.
 - **CF.MA.9-12.V.2** - Students analyze problems to determine an appropriate process for solution, and use algebraic notations to model or represent problems. (Algebraic and Analytic Thinking)
 - **CF.MA.9-12.V.2.1** - Identify important variables in a context, symbolize them and express their relationships algebraically.
 - **CF.MA.9-12.V.2.2** - Represent algebraic concepts and relationships with matrices, spreadsheets, diagrams, graphs, tables, physical models, vectors, equations and inequalities; and translate among the various representations.
 - **CF.MA.9-12.V.2.3** - Solve linear equations and inequalities algebraically and non-linear equations using graphing, symbol-manipulating or spreadsheet technology; and solve linear and non-linear systems using appropriate methods.
 - **CF.MA.9-12.V.2.4** - Analyze problems that can be modeled by functions, determine strategies for solving the problems and evaluate the adequacy of the solutions in the context of the problems.
 - **CF.MA.9-12.V.2.5** - Explore problems that reflect the contemporary uses of mathematics in significant contexts and use the power of technology and algebraic and analytic reasoning to experience the ways mathematics is used in society.
- **CF.MA.9-12.VI** - Probability and Discrete Mathematics
 - **CF.MA.9-12.VI.1** - Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available, and make critical judgments about claims that are made in probabilistic situations.
 - **CF.MA.9-12.VI.1.1** - Develop an understanding of randomness and chance variation and describe chance and certainty in the language of probability.
 - **CF.MA.9-12.VI.1.2** - Give a mathematical definition of probability and determine the probabilities of more complex events, and generate and interpret probability distributions.
 - **CF.MA.9-12.VI.1.3** - Analyze events to determine their dependence or independence and calculate probabilities of compound events.
 - **CF.MA.9-12.VI.1.4** - Use sampling and simulations to determine empirical probabilities and, when appropriate, compare them to the corresponding theoretical probabilities; understand and apply the law of large numbers.
 - **CF.MA.9-12.VI.1.5** - Conduct probability experiments and simulations, to model and solve problems, including compound events.
 - **CF.MA.9-12.VI.2** - Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying, and analyze ideas like recurrence relations, induction, iteration, and algorithm design.
 - **CF.MA.9-12.VI.2.1** - Derive and use formulas for calculating permutations and combinations.
 - **CF.MA.9-12.VI.2.2** - Use sets and set relationships to represent algebraic and geometric concepts.
 - **CF.MA.9-12.VI.2.3** - Use vertex-edge graphs to solve network problems such as finding circuits, critical paths, minimum spanning trees and adjacency matrices.
 - **CF.MA.9-12.VI.2.4** - Analyze and use discrete ideas, such as induction, iteration and recurrence relations.
 - **CF.MA.9-12.VI.2.5** - Describe and analyze efficient algorithms to accomplish a task or solve a problem in a variety of contexts, including practical, mathematical and computer-related situations.
 - **CF.MA.9-12.VI.2.6** - Use discrete mathematics concepts as described above to model situations and solve problems; and look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems) and decide upon a best solution (optimization problems).